Research Article

Mathematics Problem Solving using Metacognition Aspect

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Abstract

If students are to excel on both the routine mathematics skills and the problem-solving skills, teachers must place emphasis on both the mathematical contents and the mathematical processes in the teaching and learning of mathematics. This paper presents the theoretical rationale and the importance of metacognition to the learning of mathematics. A project was conducted on students of around sixteen years of age and the findings indicated that students did employ the four phases of problem solving emphasized by George Polya. However, students fared better when they regulated their thinking process or employed metacognitive skills in the process of solving mathematics problems. This paper also suggests the strength of a mixed methodology in doing research by expanding an understanding from one methodology to another and converging findings from different data sources.

Keywords: data mining, K map, cognitive analyses.

Introduction

Twenty-first century mathematics education is about facing novel real-world problems, nurturing creative thinking skills and cultivating productive ways of learning. In attempting to innovate teaching and learning in order to prepare a new generation for the demands of this new era, many educators have discovered the value of metacognition [1]. Mathematics is always one of the difficult subjects for school students. Von Glaserfeld (1995) says:

[Educators] have noticed that many students were quite able to learn the necessary formulas and apply them to the limited range of textbook and test situations, but when faced with novel problems, they fell short and showed that they were far from having understood the relevant concepts and conceptual relations. (p. 20)

Educators attribute the lack of mastery of metacognitive skills of our students to be one of the factors contributing to this problematic situation. A study, employing a mixed methodology by using quantitative and qualitative approaches at the same time, was conducted to investigate the effect of metacognitive skills on the problem-solving ability of students. This paper not only sheds some light on the importance of metacognition in problem solving, also elucidates the strength of a mixed methodology[10].

Problem Solving

Schools, as now organized, are a product of the industrial age. Minimum competencies in reading, writing and arithmetic were expected of all students, and more advanced academic training was reserved for the selected few. ... The educational system of the industrial age does not meet the economic needs of today. (p. 3)
four stages. Fernandez, Hadaway and Wilson (1994) provide a problem-solving model (Figure 1), which includes the managerial processes or what other educators such as Schoenfeld and Flavell called metacognition. This figure shows the non-linearity of problem solving which is actually experienced by problem-solvers. The clockwise and anti-clockwise nature of the cycle suggests that the problem-solving process can go top-down or bottom-up with reference to Pólya’s model. The managerial processes or metacognition will also trigger the problem solver to jump a stage or stages.

**Met cognition**

The concept of metacognition was first defined in the seventies. It seems that metacognition is a result of research on cognitive development, memory and reading. Many mathematics educators have shown great interest in this area as they realize that purely cognitive analyses of mathematical performance are inadequate for studying problem solving. Flavell (1976) defines metacognition as:

> Metacognition refers to one’s knowledge concerning one’s own cognitive processes and products or anything related to them, e.g., the learning-relevant properties of information or data. … Metacognition refers, among other things, to the active monitoring and consequent regulation and orchestration of these processes in relation to the cognitive objects or data on which they bear, usually in the service of some concrete goal or objective. (p. 232)

He illustrates this term further by saying that I am engaging in metacognition if I notice that I am having more trouble learning A than B; if it strikes me that I should double-check C before accepting it as a fact; if it occurs to me that I had better scrutinize each and every alternative in any multiple-choice type task situation before deciding which is the best one; if I become aware that I am not sure what the experimenter really wants me to do; if I sense that I had better make a note of D because I may forget it; if I think to ask someone about E to see if I have it right. (p. 232)

Brown (1987) says that what is of major interest is knowledge about one’s own cognition rather than cognition itself. He defines metacognition as those executive skills, which contribute to predicting, checking, monitoring, reality testing, and coordinating and controlling of deliberate attempts to learn or solve problems, and their use at the right time and in the right place.

Schoenfeld most probably has given the most comprehensive analysis of metacognition. According to Schoenfeld (1987, 1992), metacognition is thinking about our thinking and it comprises of the following three important aspects: knowledge about our own thought processes, control or self-regulation, and beliefs and intuition. The point according to Schoenfeld (1987) is that students should wisely divide their time among (a) understanding the problem, (b) planning, (c) making decisions on what to do, and (d) executing the decisions for a solution within the time frame. In the process of solving a problem, they should be monitoring and keeping track of the progress to a solution. When the decisions seem not to work, they should try other alternatives or make some adjustment. Once a decision is made to go for new alternatives, the work done should not be thrown away. There is always a risk that the curtailed efforts might have led to success [5].

Despite the apparent importance of metacognition in mathematical performance, it has not been studied systematically by mathematics educators. Several models of problem solving have been created, based on Pólya’s four-phase model, which assumes metacognitive processes only implicitly. Garofalo and Lester (1985) attribute this lack of attention to metacognition to the following three reasons:

- Covert behavior of any type is extremely difficult to observe and analyze.
- Researchers who accept self-reports as legitimate data recognize that asking a person to verbalize information while performing a task may affect the cognitive process if the verbalized information would not otherwise be attended to.
- Phenomena linked with metacognition have been regarded by many psychologists as too ill-defined for investigation [5].

**Methodology**

A study was conducted with one of its objectives to investigate the effect of metacognition on problem solving. A mixed methodology was adopted for this study that involved collecting and analyzing both quantitative and qualitative data. Researchers recognize that all methodologies have their strengths and weaknesses and feel that the strengths of one single methodology can complement the weaknesses of another methodology. In this study, the understanding gathered from quantitative analyses was expanded and elaborated through qualitative analyses. As such, this study was carried out in two parts.

Part I was predominantly quantitative. A sample of 412 students, selected randomly from a population of 2962 students, participated in this study. The instruments of this study consisted of a set of mathematics problems of 4 different levels of difficulty to be solved by the sample students for determining their problem-solving ability and a set of questionnaire to be completed by the same students for gathering students’ personal data and information related to the problem-solving process while answering mathematics problems. A factor analysis and a simple regression analysis were performed for identifying the problem-solving behavior of students.

Part II, predominantly qualitative, was carried out immediately after Part I completed. Purposive sampling was used to select 18 students, taking into consideration issues like location of the school, interest of the mathematics teachers in this research and the mathematics ability of the students. A set of mathematics problems was posed to the selected students aiming to identify their metacognitive skills while solving these problems. The answering session for each student was video-recorded separately. The answer script and any rough work were collected back. After each video recording session, the researchers met, watched the tape and discussed issues that needed clarification from the student relating to problem-solving skills and problem-solving processes employed while solving the problems. Later, the tape was replayed to the student. The researchers simultaneously conducted an interview with this student to gather more information.
on these identified issues. These interviews were taped and the audiotapes were transcribed. Scheonfeld’s (1983) episode-parsing framework was adopted to analyze the data collected for the presence of metacognitive skills or executive skills.

**Discussions and Conclusions**

No matter how students fared, the result of the factor analysis indicated that they did employ the four stages of problem solving proposed by Polya (1973). However, the mere employment of problem-solving skills is not enough to bring about success in solving mathematics problems of students. Both the value of 0.536 for $R^2$ from a simple regression analysis and the problem-solving process exhibited by Simon support this. There are three key features that caused the failure in obtaining a solution by Simon, coinciding with those proposed by Scheonfeld (1985).

The case of Angela illustrates the importance of metacognition in bringing about success in solving mathematics problems. This finding is in line with the conclusion by Scheonfeld (1985) that a good problem-solver constantly questions his or her achievement. He or she generates a number of possible candidates to the method of solution, but is not seduced by them. By making careful moves such as pursuing productive leads and abandoning fruitless paths, he or she solves the problem successfully.

**References**


